APPLICATION OF THE ENERGY CONCEPT TO THE CLIMB PERFORMANCE OF A LIGHT PROPELLOR-DRIVEN AIRPLANE

Walter C. Stewart, Jr.
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Due to the limited information appearing in the literature on the subject of energy methods, the assistance of Professor Dommasch was invaluable in locating and obtaining the material which was available.

They also wish to express their appreciation to Mr. Robert Cooper and his able assistants for their work in the maintenance of the airplane and for their aid in the instrumentation problems.

LIST OF FIGURES

SUMMARY

In this report the energy concept is applied to a low performance, propeller-driven airplane in an effort to determine maximum rate of climb and optimum energy climb schedules and to determine what advantage, if any, exists in using the latter over the former in climbing the airplane to a given energy height.

From the data obtained from acceleration runs at constant altitude a predicted schedule of time-to-climb to various altitudes is determined using both methods. This predicted result is then compared to the results obtained from actually executing the derived schedules.

It is concluded that the energy concept represents an improvement in economy of time and gasoline over the current use of sawtooth climbs to obtain maximum rate of climb data, and is equally satisfactory for that purpose. The necessity for using two pilots in the flight test operation is considered to be a minor disadvantage. This was required because this aircraft was not equipped with a constant speed propeller. In addition, it is concluded that the optimum energy climb schedule gives no significant improvement over the maximum rate of climb schedule in a low performance aircraft of this type.

This investigation was conducted in partial fulfillment of the requirements for the degree of Master of

Science in Engineering by Major Walter C. Stewart, Jr., USMC, and Lieutenant George H. Hughey, Jr., USN, during the 1953-1954 academic year at the Forrestal Research Center, Princeton University, Princeton, New Jersey.

APPLICATION OF THE ENERGY CONCEPT TO THE CLIMB PERFORMANCE OF A LIGHT PROPELLER-DRIVEN AIRPLANE

INTRODUCTION

The climb performance of an airplane is customarily described in terms of its maximum sea level rate of climb, its service ceiling and its time to climb to a given altitude. In the past, the speed range of the airplane being small, the change in kinetic energy was negligible in a climb, and a climb essentially consisted of using the power plant energy to change the potential energy of the airplane.

As the performance of aircraft improved, the climbing speeds increased and the variation between maximum and minimum speeds became greater. Thus the changes in kinetic energy in the transition from take-off to climb, during the climb, and during the acceleration from climb to level flight became important since they too required the expenditure of energy by the power plant.

The total energy stored in an aircraft may be defined as the sum of its potential and kinetic energies. These two forms of energy are readily interchangeable in flight since a dive converts potential to kinetic energy and an

abrupt pull up from level flight converts kinetic to potential energy. It is therefore evident that for a given altitude (i.e. potential energy level) an airplane may have any velocity (i.e. kinetic energy level) within its capacity. Thus the airplane may have a wide range of values for its total energy at a given altitude, depending on its kinetic energy.

Now, except by arbitrary definition, the problem of climbing an airplane is not, and never has been, one of proceeding from one altitude to another in a minimum period of time. The airplane must perform some useful function upon arriving at the desired altitude, whether it be cruising, maneuvering or fighting. All of these functions depend directly upon arriving at a given combination of altitude and airspeed in a minimum period of time. The advent of high performance aircraft has merely aggravated this problem, not changed it. From this it may be concluded that altitude alone is not an adequate criterion of climb performance. This has led to the concept of "energy level" as a more realistic basis upon which to investigate climb performance.

The total energy of an airplane in flight may be expressed by the equation

$$E = PE + KE$$
$$= Wh + \frac{WV^2}{2g}$$

Since the items of primary interest are altitude and air-speed, the weight may be eliminated

$$\frac{E}{W} = h + \frac{V^2}{2g} \tag{2}$$

and if "specific energy" is defined as the energy per unit weight of the airplane, the foregoing equation becomes one of specific energy. However, noting each term has units of length and thus may be thought of as an equivalent altitude, the more descriptive term "energy height" (h_e) has been introduced to replace specific energy. Thus,

$$h_e = h + \frac{V^2}{2g} \tag{3}$$

The problem of determining the speed for maximum rate of storing potential energy (maximum rate of climb) is thus replaced by the problem of determining the speed for maximum rate of storing total energy (optimum energy climb). This reduces directly to the determination of a speed schedule for the maximum rate of change of energy height.

While, from the foregoing, it is apparent that the energy concept may be a useful approach to the investigation of a high performance aircraft, the authors became interested in the feasibility of applying it to a low-performance, propeller-driven aircraft. The current practice of employing saw-tooth climbs to determine the climb

performance of aircraft is time consuming, expensive, difficult and subject to considerable inaccuracy. It has, however, been the only method in common use.

The energy concept, while not new (See Bibliography) has received but little attention in the literature, and its application to actual flight testing has been limited. At the suggestion of Professors C.D. Perkins and D.O. Dommasch of the Department of Aeronautical Engineering, Princeton University, the authors undertook to use level flight acceleration runs as a means of determining maximum rate of climb and optimum energy climb schedules for a North American "Navion", single-engine, low-wing, light plane, powered with a Continental 205 horsepower engine. These schedules were then flown in an effort to determine whether either schedule gave appreciably superior results in time to climb to a given energy height. In addition, certain arbitrary schedules were flown for comparison purposes.

THEORETICAL DEVELOPMENT

As discussed in the introduction, the energy concept involves the shortest time to climb to a desired energy level or energy height. It is apparent then that a climb schedule must be developed such that at each energy height attained, the rate of increase of energy height is as large as possible. This climb schedule will be called the optimum energy climb schedule. Since the pilot is interested in the relation of airspeed to altitude, the climb schedule must be presented in the form

$$V = f(h)$$

To begin, certain assumptions must be made as follows:

(1) The climb occurs in a standard atmosphere.

(2) The climb is made in a single vertical plane.

(3) There is no atmospheric turbulence or wind gradient.

(4) The engine power settings are either constant or are made according to some fixed schedule.

Again writing equation (3)

$$h_e = h + \frac{V^2}{2g} \tag{3}$$

Then by differentiation of equation (3), we obtain the rate of change of energy height, which will be designated by the letter "w"

$$w = \frac{dh_e}{dt} = \frac{dh}{dt} + \frac{1}{2g}\frac{dV^2}{dt} \tag{4}$$

Consider two energy heights, h_{e_1} and h_{e_2}, with $h_{e_2} > h_{e_1}$. Since $w = \frac{dh_e}{dt}$, $dt = \frac{dh_e}{w}$, and the time to climb from h_{e_1} to h_{e_2} becomes

$$t = \int_{h_{e_1}}^{h_{e_2}} \frac{dh_e}{w} \quad . \tag{5}$$

In order to minimize the time, it is necessary to minimize the integral. Therefore it is seen that w needs to be a maximum for the interval between h_{e_1} and h_{e_2}. This is not necessarily the maximum w that could be obtained at a particular altitude, as will be seen later.

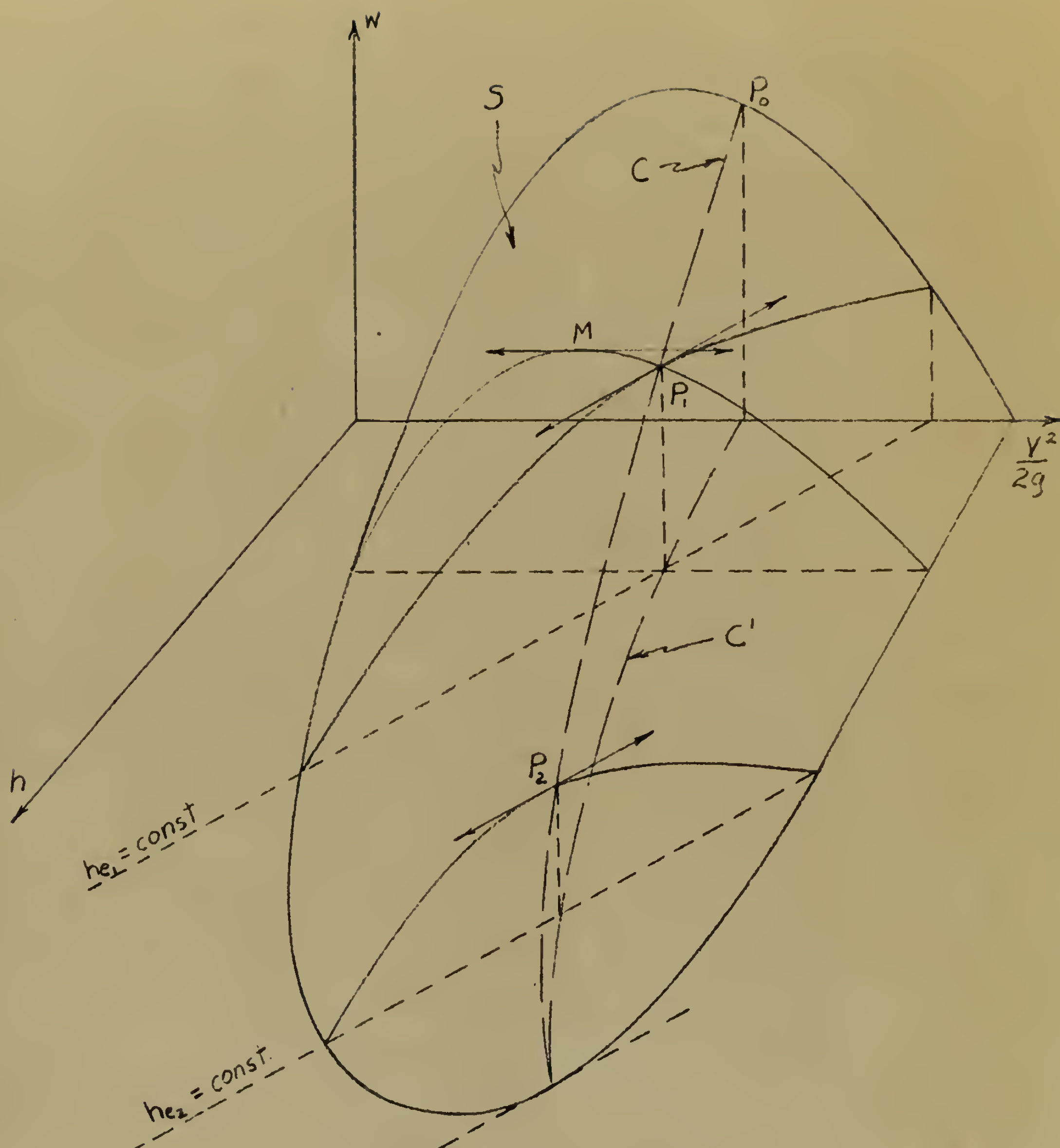
Chapter 7, Part II of Ref. 6 illustrates a simple geometric method of finding the speeds at various energy heights for which w is a maximum. Since equation (4) shows that

$$w = f(h, \frac{V^2}{2g})$$

a three- dimensional graph could be drawn with w, h and $\frac{V^2}{2g}$ as coordinates. The resulting figure would be a surface (S) which is the locus of all points satisfying equation (4). Sketch 1 shows how this figure would appear.

Before explaining the significance of Sketch 1 in the determination of the optimum energy schedule, the methods of experimentally finding surface (S) should be

discussed.



Sketch 1

There are two methods which can be used to find surface (S):

(1) By level flight acceleration runs.

(2) By continued climbs following schedules in

the vicinity of the supposed optimum energy schedule.

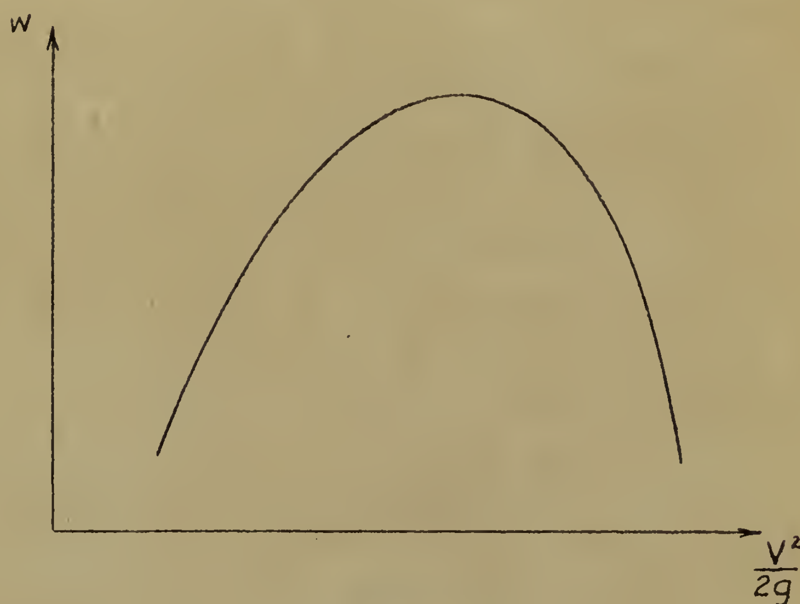
The level flight acceleration run method considers the fact that the excess power of an airplane may be used to produce a rate of change of potential energy, kinetic energy, or both. In a climb at constant true airspeed the only change is in potential energy, while in a level acceleration run the only change is in kinetic energy. So for each altitude at which an acceleration run is made, the change in total energy will be a function of velocity only. Or, in terms of energy height

$$\frac{dh_e}{dt} = \frac{d}{dt}\left(\frac{V^2}{2g}\right) \tag{6}$$

From the velocity and time data obtained in the acceleration run the type of graph illustrated in Sketch 2 may be obtained for each altitude.

Sketch 2

If the curve in Sketch 2 is graphically differentiated and divided by 2g, the result may be plotted as $\frac{d}{dt}\left(\frac{V^2}{2g}\right)$ or w versus $\frac{V^2}{2g}$. The result would be as shown in Sketch 3.



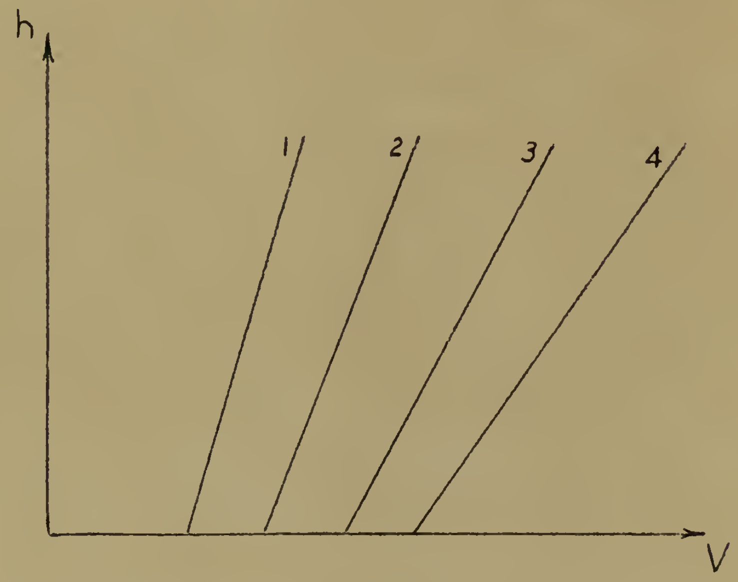
Sketch 3

Curves of the type shown in Sketch 3 can be made for various altitudes and if these curves are plotted on the axis system of Sketch 1, the result will be surface (S).

It may also be noted that the acceleration run curves have another important feature. Since either rate of change of potential energy at constant velocity or rate of change of kinetic energy at constant altitude will be a maximum at the speed for maximum excess power, the point of inflection on the curve of Sketch 2 or the maximum point of the curve of Sketch 3 determines the speed

for the so-called best rate of climb (maximum rate of change of potential energy) for the altitude at which the acceleration was made.

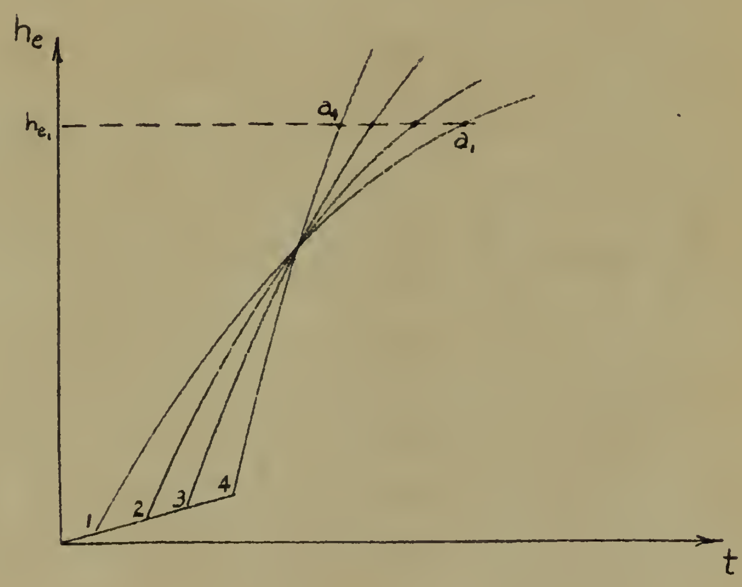
If the continued climb method is used, several timed climbs should be made to ceiling at various schedules of velocity versus altitude during which the true airspeed is linearly increased with altitude. These schedules are as shown in Sketch 4.



Sketch 4

It is necessary to assume a standard lapse rate in order to convert these schedules to schedules in terms of observed airspeed versus observed pressure altitude for the benefit of the pilot who will fly the schedules. Even though the actual data taken in flight, after reduction to standard condition, may not exactly corres-

pond to the desired schedule, the difference will not affect the evaluation of the results. The reason that this is true is; first, that any deviations from the standard lapse rate will be small and thus will not greatly affect the airspeed or altitude, and second, some useful schedule of velocity versus altitude will be obtained even though it may not be exactly the prescribed schedule. The corrected data can then be used to plot curves of h_e as a function of time as in Sketch 5.



Sketch 5

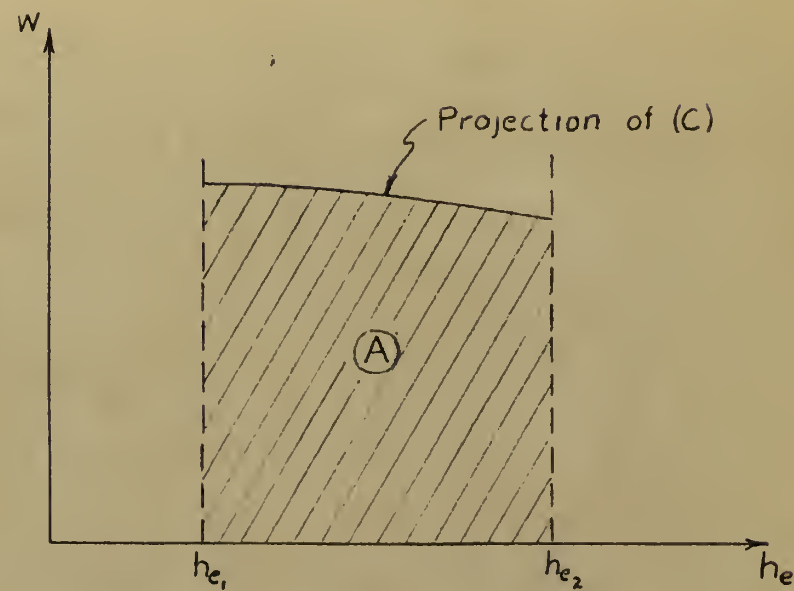
By graphical differentiation, values of $\frac{dh_e}{dt}$ or w can be found for each value of time. Curves of h and $\frac{V^2}{2g}$ as functions of time can also be plotted from the corrected test data. Corresponding values of w, h and

$\frac{V^2}{2g}$ can then be used to form surface (S).

Now that the methods of determining the surface shown in Sketch 1 has been established, we may proceed to investigate its significance. As has been previously stated, the object is to find the maximum w in the interval between h_{e_1} and h_{e_2}. If the plane of h_{e_1} = constant is established in Sketch 1, the intersection of this plane and surface (S) will be a curve w = f(V) with h_e constant. The maximum point on this curve will be determined by the tangent line which lies in the plane of h_{e_1} = constant and which is parallel to the plane w = 0. The same method is used to establish the maximum w for h_{e_2} = constant and for all the other h_e planes which can be drawn through the figure. Curve (C) on the surface (S) is the locus of all these points of maximum w and is therefore the optimum energy climb schedule. Since the pilot is interested in some schedule V = f(h), curve (C) can projected on the plane w = 0 to establish curve (C'), the practical definition of the optimum energy climb schedule.

In order to show that curve (C) is the locus of points which satisfy the condition of maintaining the maximum rate of change of energy height, the curve may be projected onto the plane $h = \frac{V^2}{2g}$. This is the plane which is perpendicular to all of the h_e = constant planes.

This projection would appear as the graph shown in Sketch 6.



Sketch 6

This graph shows that the area (A) between h_{e_1} and h_{e_2} will be a maximum when the curve $w = f(h_e)$ is the projection of curve (C). Therefore curve (C) must represent the optimum energy climb schedule.

It is now desirable to inspect the figure in Sketch 1 and analyze its properties. They may be listed as:

(1) The projection (C') of the curve (C) on the plane $w = 0$ expresses the optimum energy climb schedule in the practical form

$$V = f(h)$$

(2) The figure shows that if P is the maximum of the $w = f(V)$ obtained with h_e = constant, and if M is the maximum of the curve $w = f(V)$ obtained for h = constant,

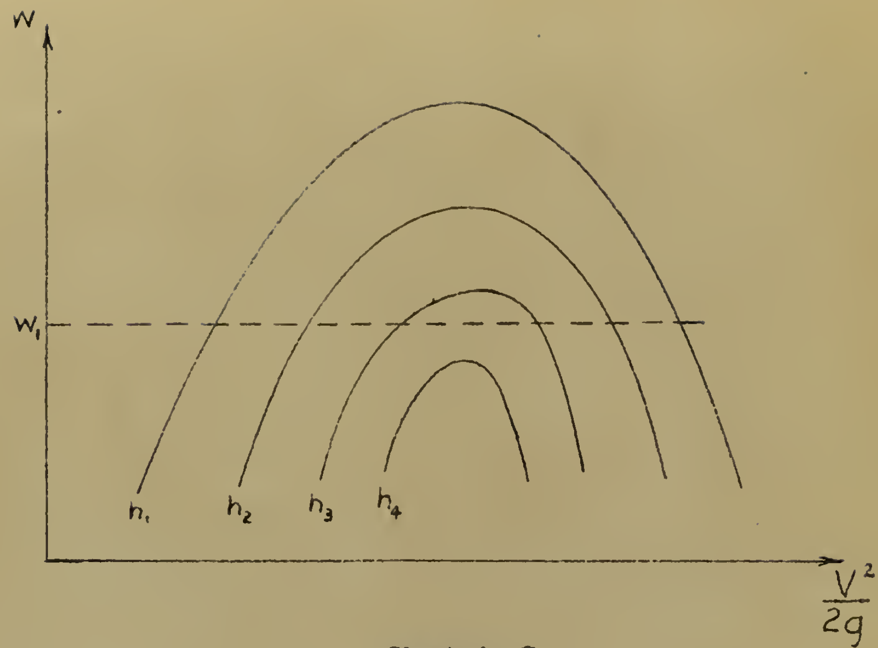
then the point P corresponds to a greater speed than that of M.

(3) Point M corresponds to the speed for best rate of climb (maximum rate of change of potential energy).

(4) In the vicinity of the optimum climb schedule w does not vary appreciably from its maximum value along the curve formed by the h_e = constant plane and surface (S). Therefore small variations in speed from the optimum schedule will not have much effect on the time to climb.

Although the optimum energy schedule has now been established by the figure in Sketch 1, it is obvious that it would be rather difficult to construct a three-dimensional graph and attempt to get accurate values from it. It is possible, however, by various schemes of cross-plotting the test data to present equivalent two-dimensional figures from which the required optimum energy climb schedule can be found.

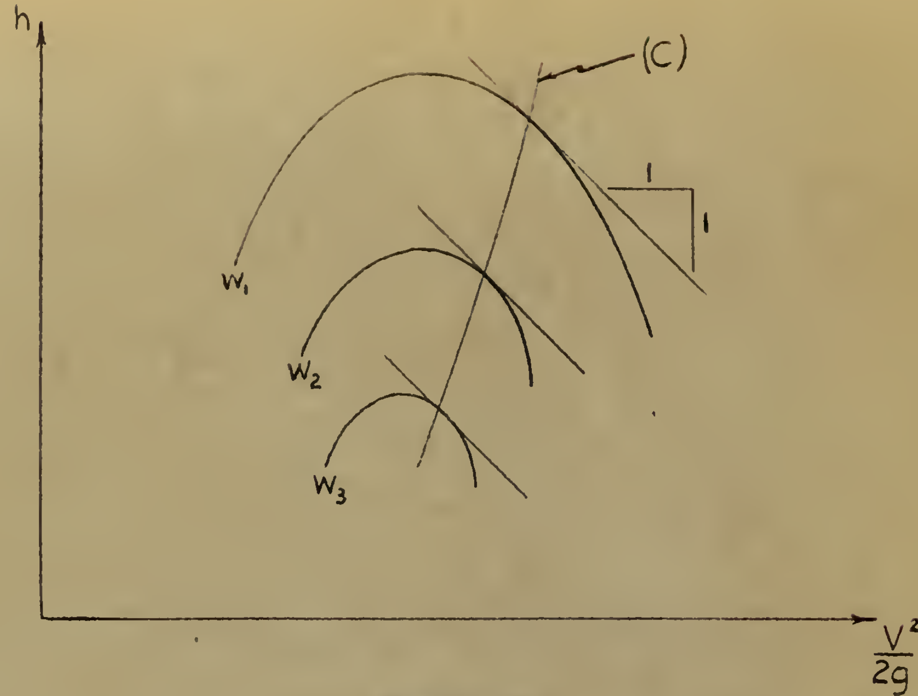
The first method uses the data obtained from the level flight acceleration runs. It has previously been demonstrated how the acceleration run for each altitude could be plotted in terms of w versus $\frac{V^2}{2g}$ as in Sketch 3. If these curves were to all be plotted on the same graph the result would appear similar to that shown in Sketch 7.



Sketch 7

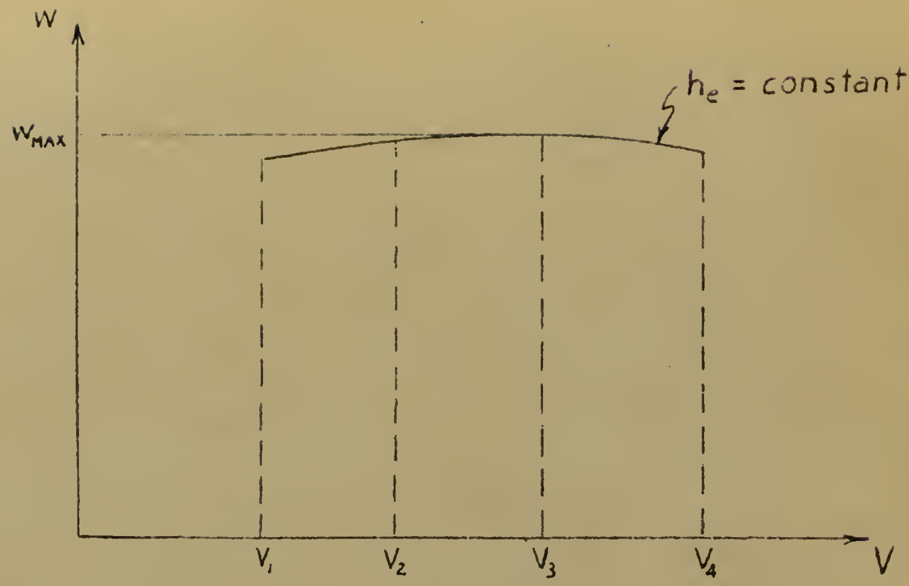
The next step is to construct the cross sections w = constant by cross-plotting the data of Sketch 7. This presentation is demonstrated by Sketch 8.

Thus the intersections of planes of constant w with the surface (S) are projected on the w = 0 plane of Sketch 1. The slope of the line h_e = constant in this plane is given by the normal to the bisector of the axis system. This will be a line with negative 45° slope if the same scale is used for both the abscissa and the ordinate. Tangents to the curves of Sketch 8 which are parallel to the h_e = constant lines will determine points of the optimum energy curve (C).



Sketch 8

Another method of determining the optimum energy climb schedule may be used with the data obtained from the continued climbs. Referring to Sketch 5, lines of constant h_e may be drawn. At these intersections, such as a_1, a_2, a_3 and a_4 for constant h_{e_1}, the slopes of the curves may be found for different values of V. Since these slopes are $\frac{dh_e}{dt}$ or w, they may be plotted as a function of V as shown in Sketch 9. From this graph, the optimum speed may be found for the selected value of h_e. By constructing similar graphs for various values of h_e, the climb schedule can be found.



Sketch 9

It will be noted that for both these methods, the value of w can also be found for each combination of altitude and velocity. This combination will of course determine a value of h_e and so the relation $w = f(h_e)$ can be found. From this the time to climb can be found by direct integration. In practice, however, it is best to verify the schedule by an actual climb.

The foregoing discussion has been only one version of the subject of energy climb methods and has been somewhat brief. Other authors have given more rigorous analyses of the problem but it is believed that the geometric solution used in this thesis and given by Chapter 7, Part II of Ref. 6 is the easiest and most direct method of finding the energy climb schedule.

EQUIPMENT AND PROCEDURE

This investigation was undertaken with two primary purposes in view; first, to determine the feasibility of applying the energy concept to the determination of climb schedules for a low performance, propeller-driven aircraft, and second, to determine whether any significant improvement in performance could be obtained using the optimum energy climb concept as opposed to the maximum rate of climb concept on such an aircraft. In view of the first condition, the following description of the procedure used will be somewhat more detailed than might normally be considered appropriate for an investigation of this type.

Figure 1 illustrates the aircraft employed which was a North American "Navion" low wing, single-engined monoplane powered with a Continental 205 horsepower engine. No significant changes in the production model had been made except a reduction in the span of the horizontal tail, which was done for purposes not connected with this investigation.

All data, with the exception of temperature, was recorded from a photo-panel, illustrated in Figure 2, mounted just aft of the co-pilot's seat, which contained the following instruments:

(1) Sensitive altimeter

(2) Sensitive airspeed indicator

(3) Manifold pressure gauge

(4) Tachometer indicator

(5) Stopwatch

(6) Dual servo indicator (not used)

The camera in this recorder was a modified Fairchild GSAP gun camera with a speed of two frames per second, which had focusing and aperture adjustments.

The altimeter and airspeed indicator were calibrated against laboratory standards for instrument error and the results were applied to all recorded data as necessary.

Both pitot and static pick-ups were located on free-swiveling heads on a wing boom on the starboard wing as shown in Figure 1. It was consequently anticipated that there would be little or no position error in either the airspeed or altimeter readings. This proved to be the case when the position error was determined by the Tower Method, which consists, briefly, of comparing the readings of two altimeters at the same corrected altitude, one in an airplane moving at various airspeeds, and one stationary in a tower. The difference between the two readings at each airspeed is presumed to be the position error of the moving instrument at that airspeed. If it is assumed that all of the position error of the airspeed indicator is in the static line and none in the pitot line (a reasonable assumption), then the foregoing may be used directly to determine the position error of the airspeed

indicator. This information is plotted in Figures 3 and 4.

Due to the low accelerations anticipated, and the lack of sufficient equipment with which to determine lag errors, no compensation for instrument lag was undertaken. It is not considered that this omission introduced significant errors.

Four consecutive acceleration runs were then made at each of the following pressure altitudes; 1,000 ft., 4,000 ft., 7,000 ft., and 10,000 ft., a total of sixteen runs in all. This was done in order to check the reproducibility of the results, one of the desired purposes of the investigation.

The "ceiling" of 10,000 feet was chosen arbitrarily due to the excessive time which would have been involved in a climb to the true service ceiling.

These runs were conducted in the following manner. The airplane was trimmed in a full-throttle, 2300 rpm climb, at an altitude about three hundred feet below the test altitude, with flaps down, at a speed of from seventy to eighty miles per hour. On arriving at the test altitude the airplane was leveled off, the flaps retracted, and the airplane allowed to accelerate to V_{max}, while the altitude and power settings were held constant. Since the plane was equipped with a controllable pitch propeller the rpm was maintained constant manually by the co-pilot.

During the acceleration runs, continuous photo-recordings were made of the time and velocity readings, and a record of the indicated ambient air temperature was made by the pilot. In addition, the manifold pressure and rpm were photorecorded in order to permit due consideration to be given to power variations during the run. From this information it was possible to plot V^2 versus t as shown in Figures 5, 6, 7, and 8.

After reduction of the test data, maximum rate of climb and optimum energy climb schedules were determined by methods described under Theoretical Development.

By plotting the maximum rate of climb and optimum energy climb schedules on Figure 9 it was possible to construct a cross plot of 1/w versus h_e for both schedules as shown in Figure 13. Since $t = \int_{h_{e_1}}^{h_{e_2}} dh_e/w$, graphical integration of this plot then gave a theoretical time to climb to any given energy height, based only on the acceleration run data. By plotting this data as h_e versus t in Figure 14, a direct comparison was obtained between the predicted and actual results.

Since the foregoing schedules, shown in Figure 11, were in terms of V versus h, it was necessary, for the pilot's benefit, to convert them to V_o versus h_{p_o}. This necessitated certain assumptions and techniques as follows. Standard temperature and pressure lapse rates were assumed

since it would be exceedingly difficult to correct for variations therefrom in a continuous climb, and since the errors introduced were negligible for the purposes of this investigation as will be shown. Then, assuming a standard atmosphere, V and h were converted to V_o and h_{p_o} and plotted as shown in Figure 12. An identical plot of the schedules was then made on a translucent sheet of paper to be used as an overlay on the original plot. Thus, to account for variations from standard initial conditions (but still assuming standard lapse rates) it was only necessary to determine the standard temperature for the pressure altitude existing at the take-off point, and the actual temperature at that point. Thus, using the standard temperature, the "standard" density altitude was determined, and, using the actual temperature, the actual density altitude was determined. From these density altitudes the density ratios σ_s and σ were determined. The corrected V_o was then determined from:

$$V_o = V_{o_s} \frac{\sqrt{\sigma}}{\sqrt{\sigma_s}}$$

where:

V_o = the observed velocity to be actually used in flying the schedule

V_{o_s} = the "standard" observed velocity appearing on the basic plot of the schedule

σ = the density ratio corresponding to the actual density altitude

σ_s = the density ratio corresponding to the "standard" density altitude.

With the assumed lapse rates, $\sqrt{\sigma}/\sqrt{\sigma_s}$ and hence $V_o - V_{o_s}$ would be approximately constant with altitude for the small speed range involved. Hence it was only necessary to move the overlay horizontally from V_{o_s} to V_o at the field pressure altitude and fly the schedule so determined using the coordinates of the original plot.

This was accomplished with considerable precision by having the co-pilot observe the altimeter and continuously call the desired airspeed to the pilot. At the same time the co-pilot recorded time and temperature versus pressure altitude every thousand feet. Then, by graphical integration, it was possible to determine tape-line altitudes, and from this, energy height versus time. This plot of h_e versus t is shown in Figure 14.

Additional arbitrary schedules as shown in Figure 12 were also flown in order to test the accuracy of the hypothesis that the optimum energy climb schedule was indeed "optimum".

RESULTS AND DISCUSSION

In the following discussion it is to be noted that no attempt has been made to account for non-standard engine thrust resulting from unsteady air flow during acceleration or from varying propeller efficiency. Since the speed range and acceleration were both small it is not considered that this has introduced significant errors in the analysis.

The reproducibility of the results of the acceleration and climb-schedule run is, as has been mentioned, of considerable importance.

It was anticipated that a light aircraft might be unduly affected by wind gradients and turbulence. Both of these factors did, in fact, prove to be important. In rough air, the acceleration-run data obtained was very unsatisfactory and the resultant curves were largely determined by the French curve employed. Horizontal wind gradients appeared to cause occasional abrupt breaks in the continuity of the curves but this was not particularly troublesome since these breaks were readily apparent and could be compensated for with permissible accuracy by judicious fairing or replotting. The curves selected are shown in Figures 5, 6, 7 and 8.

It is considered, however, that this method is

still superior in this respect to the sawtooth climb method, since the latter is relatively more sensitive to turbulence.

In relatively calm air, however, the curves obtained were smooth and reproducible to a close approximation. On the other hand the slopes of these curves in the vicinity of the inflection points had a very slight rate of change and it was necessary to use extreme care, even with the optical differentiator used, in determining them. However, such care yielded satisfactory curves of w versus $V^2/2g$ as shown in Figure 9.

As previously mentioned, the maximum ordinates of these curves may be used to determine the maximum rate of climb schedule. However, both this schedule and the optimum energy climb schedule may be more readily determined from a plot of h versus $V^2/2g$ such as Figure 10. Cross plotting of Figure 9 yielded Figure 10 directly.

As was anticipated, the maximum rate of climb and the maximum energy climb schedules determined from Figure 10 showed very little difference. This is probably due to the fact that, in the vicinity of the maximum excess power region on the airplane performance charts, the power available and power required curves are approximately parallel over a considerable range. This, however, is a characteristic of the aircraft and illustrates no defect

in the procedure. It does indicate that the maximum energy schedule would have no pronounced advantage over the maximum rate of climb schedule as will be demonstrated by the results of the continuous climbs.

The maximum rate of climb and optimum energy climb schedules were replotted for convenience in Figure 11 using an enlarged scale.

In executing the climb schedules it was found to be a simple matter to hold the airplane within one or two miles per hour of the schedule and to record time, temperature and altitude with sufficient accuracy. No noticeable turbulence or wind gradients were encountered and their effect is therefore unknown but is presumed to be much less than in the case of the acceleration runs, due to the much greater time involved and the much smaller speed range covered.

The results of the continued climbs were plotted in terms of energy height versus time in Figure 14. These results show that both the optimum energy climb and the maximum rate climb were nearly the same and closely approximated the theoretical schedule. The slight drop of the optimum energy schedule below the maximum rate schedule which occurred over part of the climb might be due to slight power variations. Since the propeller control was manually operated by the

pilot to maintain 2300 rpm, the engine speed varied slightly during the climbs. In addition, since there was no rudder trim control in this airplane, it was necessary for the pilot to maintain a continuous rudder force throughout the climb to maintain zero sideslip. Any relaxation of this force would have caused a slight additional drag which would affect the climb results.

The two additional climb schedules shown in Figure 14 indicate that they gave lower energy climb rates than the optimum energy schedule, as was anticipated. It is noted however that the 100 mph observed airspeed schedule closely followed the optimum energy schedule over approximately three-fourths of the energy height range. This is due to the fact that 100 mph is an approximate mean of the optimum energy velocities in this range. Therefore, it might be stated as a "rule of thumb" for this airplane that a constant observed airspeed of 100 mph would closely approximate the optimum energy schedule up to a pressure altitude of 10,000 feet.

It may be recalled that a conclusion was drawn in the Theoretical Development to the effect that small variations in airspeed from the optimum energy schedule would have little effect on the climb. The results in Figure 14 tend to substantiate this.

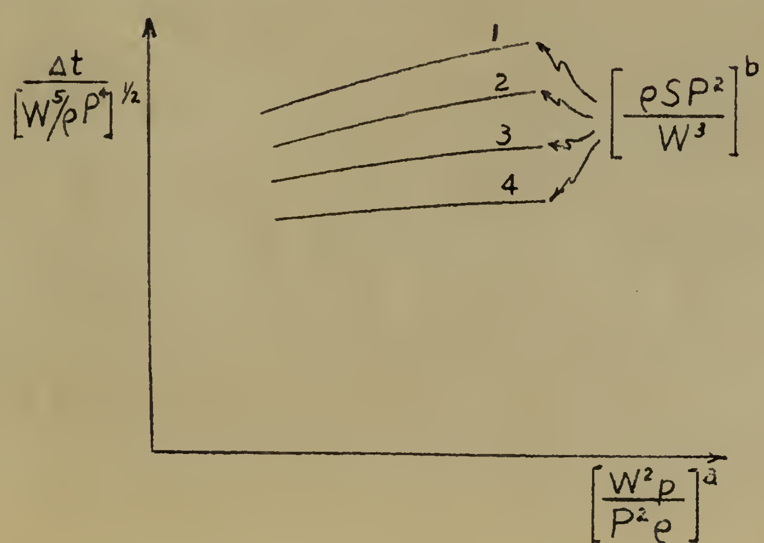
As suggestions for further investigation, the following are presented as semi-empirical approaches to the energy concept.

In the Theoretical Development it has been indicated that the advantage of the optimum energy climb over the maximum rate of climb in the time required to climb to a given energy height is greatest for a high performance aircraft. It has further been established that for a low performance aircraft the advantage is negligible.

If it is assumed that this advantage (Δt) is a function of W, ρ, p, S and P, and dimensional analysis is employed, an empirical relationship of the following form is obtained.

$$\Delta t = \left[\frac{W^5}{\rho P^4}\right]^{\frac{1}{2}} \left[\frac{W^2 p}{P^2 \rho}\right]^a \left[\frac{\rho S P^2}{W^3}\right]^b$$

This may be plotted in the following manner using non-dimensional parameters.



From the foregoing plot, and using additional data from test aircraft on which energy analyses have been conducted, a useful relationship may be established upon which to base a predicted Δt for any aircraft.

Similarly, if w is assumed to depend upon the same parameters, W, ρ, p, S and P, the following relationship is obtained.

$$w = \frac{P}{W}\left[\frac{W^2 p}{P^2 \rho}\right]^c \left[\frac{\rho S P^2}{W^3}\right]^d$$

This may be treated in like manner, but it is to be noted that a more fundamental relationship exists, namely:

$$w = \frac{TV - DV}{W}$$

and if both sides are divided by P/W:

$$\frac{w}{P/W} = \frac{TV - DV}{WP/W}$$

or:

$$w = \frac{P}{W}\left(1 - \frac{D}{T}\right)$$

where:

T = thrust
D = drag

Further investigation of the foregoing relationships is indicated.

CONCLUSIONS

It is concluded that the acceleration run method of determining energy data is entirely satisfactory for an aircraft of this type, and is, in addition, much more economical of time and gasoline than sawtooth climbs for obtaining maximum rate of climb schedules.

It is considered, however, that a minor drawback is the necessity of employing two pilots to obtain the data due to the necessity for maintaining an approximately constant power setting of the engine and of recording data, such as outside air temperature, which is not readily recorded in a photopanel. This would probably be characteristic of any lightplane so tested.

It is further concluded that the optimum energy climb schedule does not give a significant improvement over the maximum rate of climb schedule in a low performance aircraft of this type.

In addition small variations of speed from the optimum energy schedule will have negligible effect on the results.

BIBLIOGRAPHY

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2. Hamlin, Benson, <u>Flight Testing Conventional and Jet Propelled Airplanes</u>, Macmillan, New York, 1946.
3. Perkins, Courtland D. and Hage, Robert E., <u>Airplane Performance Stability and Control</u>, Wiley, New York, 1949.
4. Fuhrman, R.A., <u>Report on Application of the Energy Concept to the Climb Performance of Turbo-Jet Propelled Airplanes</u>, Test Pilot Training Division Investigative and Development Project No. 2, Naval Air Test Center, Patuxent River, Md., 24 March 1952.
5. Rutowski, Edward S., <u>Energy Approach to the General Aircraft Performance Problem</u>, Journal of the Aeronautical Science, Vol. 21, March 1954.
6. <u>Flight Testing, Vol. I (Performance)</u>, Advisory Group for Aviation Research and Development, North Atlantic Treaty Organization, edited by D.O. Dommasch, (to be published).

FORMULAS AND SAMPLE CALCULATIONS

Reduction of V_o to V at 1000 feet.

1	2	3	4	5
V_o	V_i	V_{cal}	V_e	V
80	79	77	77	78

Column

1 Observed value.

2 $V_o + \Delta V_o = V_i$. (ΔV_o, the instrument error correction, was a constant of -1 mph for the instrument used.)

3 $V_i + \Delta V_i = V_{cal}$. ($\Delta V_i$, the position error correction, was obtained from Figure 4).

4 $V_{cal} + \Delta V_c = V_e$. ($\Delta V_c$, the compressibility error correction was negligible for the low velocities encountered.)

5 $V_e/\sqrt{\sigma} = V$. ($\sqrt{\sigma}$ at 1000 is .9854.)

Reduction of h_{p_o} to h

1	2	3	4
h_{p_o}	h_{p_1}	h_p	h
1035	1010	1000	1025

Column

1 Observed value.

2 $h_{p_o} + \Delta h_{p_o} = h_{p_1}$. (Δh_{p_o}, the instrument error, was obtained from the calibration table for the instrument used).

3 $h_{p_1} + \Delta h_{p_1} = h_p$. (Δh_{p_1}, the position error, was obtained from Figure 3).

4. $h = \int_{h_{p_o}}^{hp} \frac{T}{T_s}\, dhp = \sum_{h_{p_o}}^{hp} \frac{T}{T_s} \Delta h_p$. (This was obtained by graphic integration of a graph of T/T_s versus h_p.) (See Ref. 6).

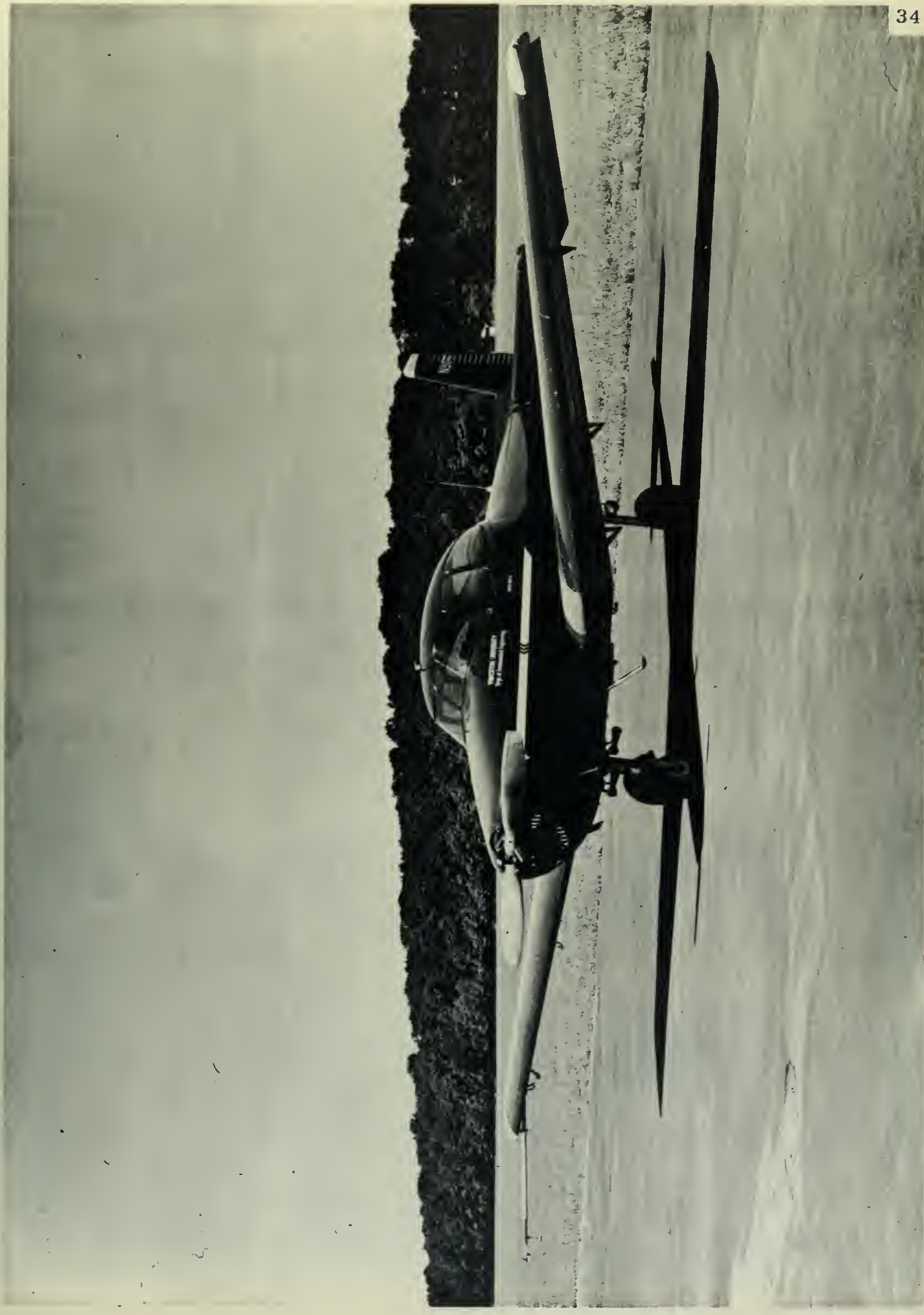

Figure 1

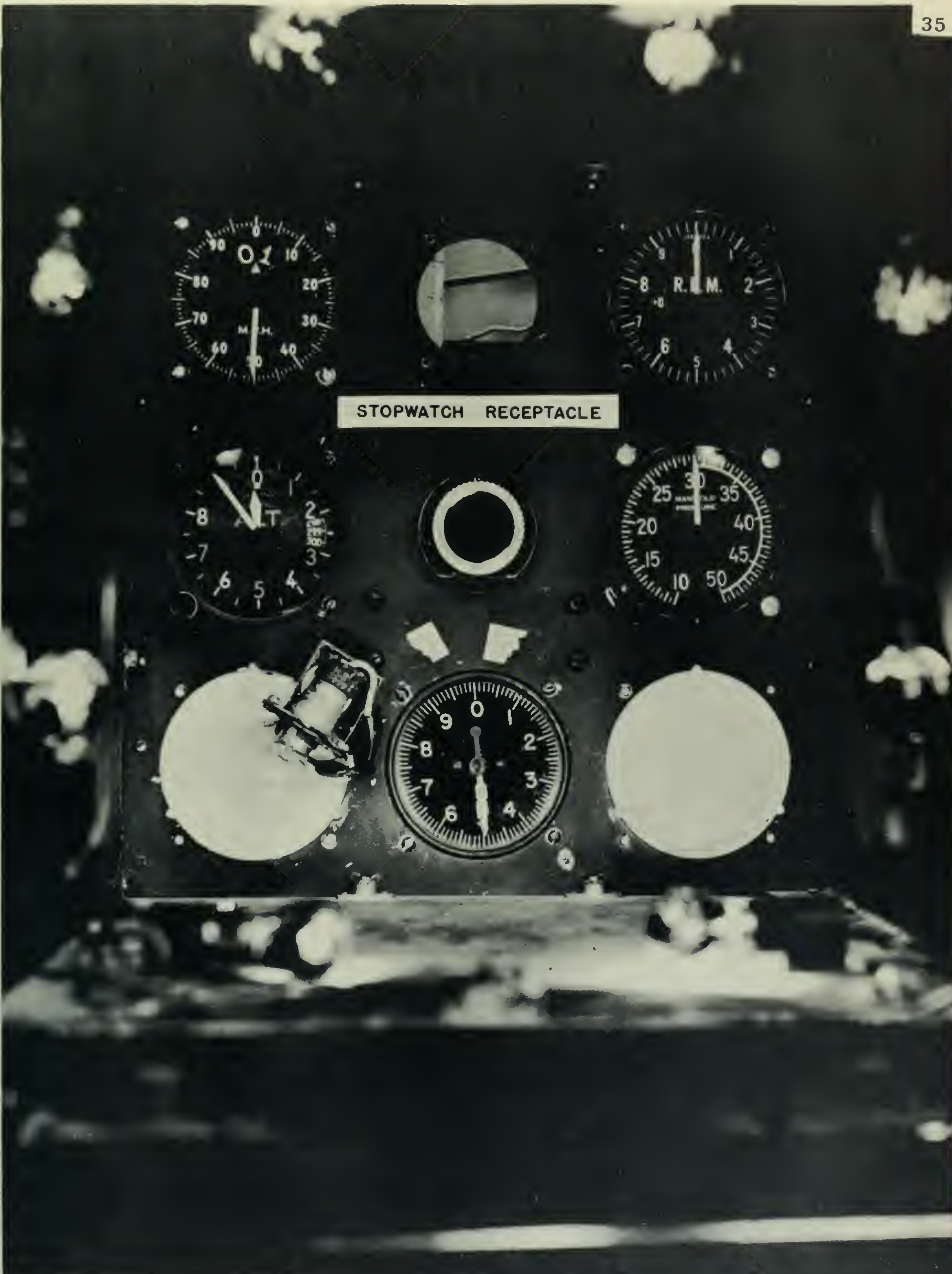

Figure 2

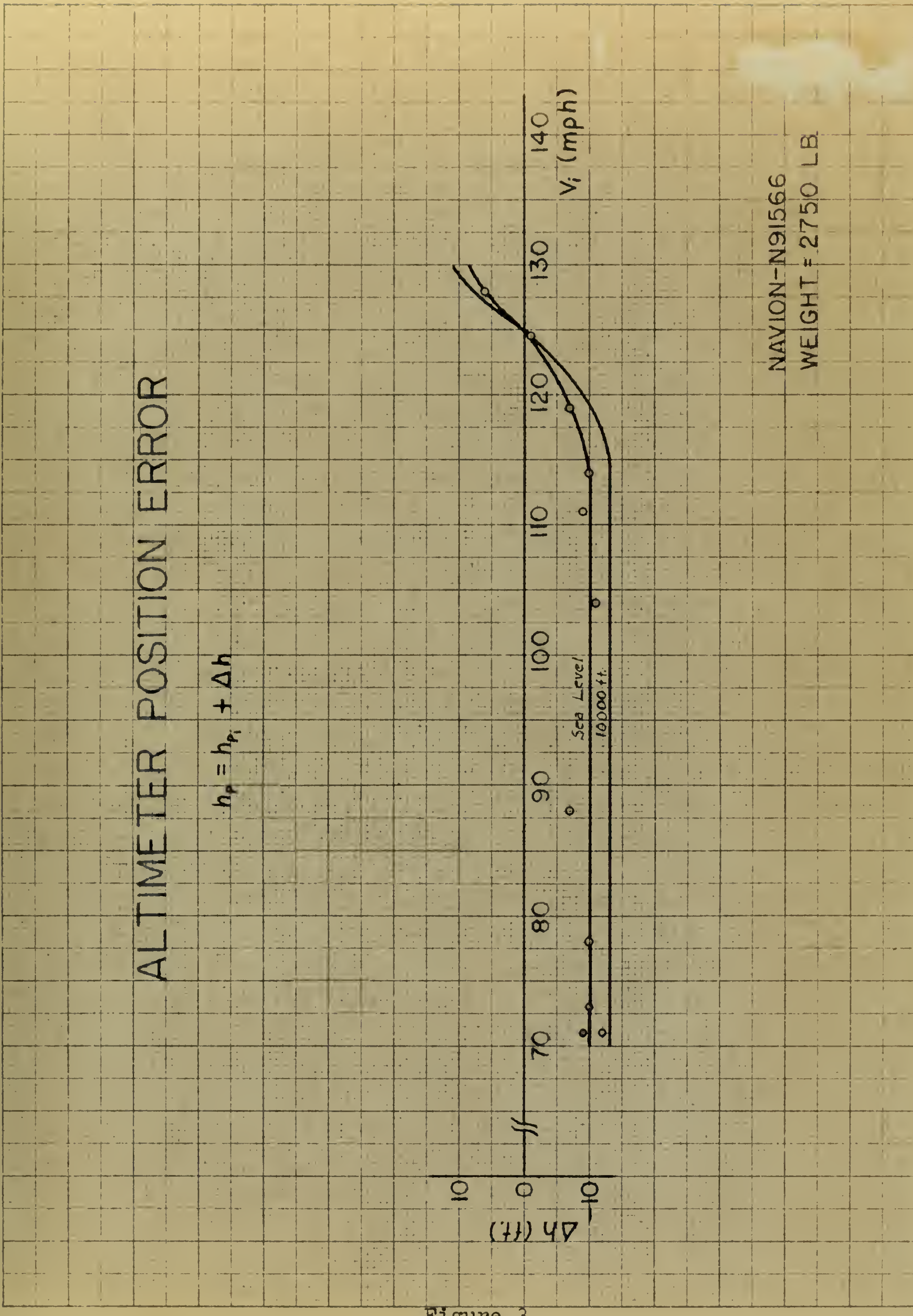
ALTIMETER POSITION ERROR
$h_p = h_{p_i} + \Delta h$
NAVION-N91566
WEIGHT = 2750 LB
V_i (mph)
Δh (ft)
Sea Level
10000 ft.
70
80
90
100
110
120
130
140
10
0
-10

Figure 3

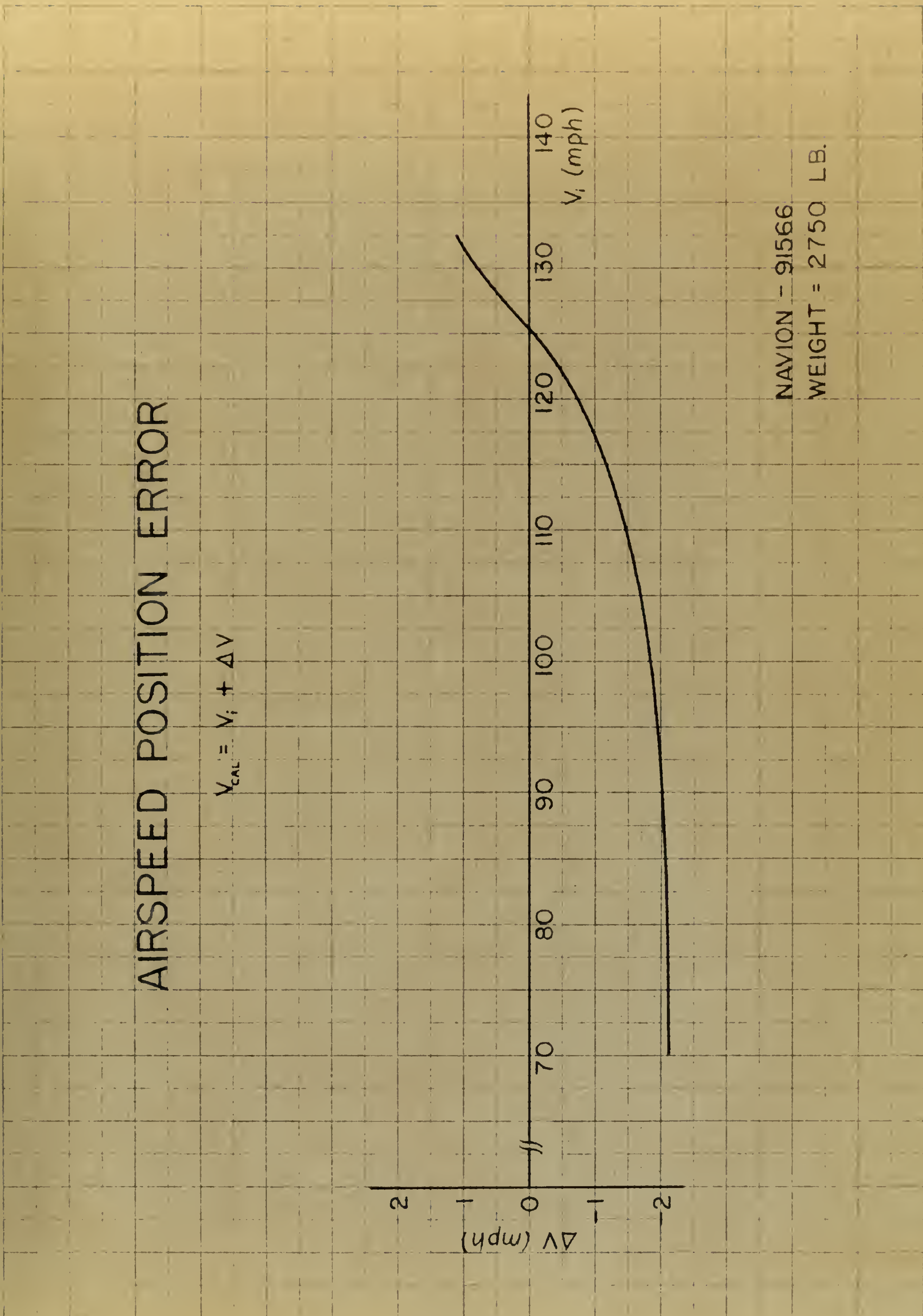
AIRSPEED POSITION ERROR
$V_{CAL} = V_i + \Delta V$
NAVION - 91566
WEIGHT = 2750 LB.
V_i (mph)
70
80
90
100
110
120
130
140
ΔV (mph)
2
1
0
-1
-2

Figure 4

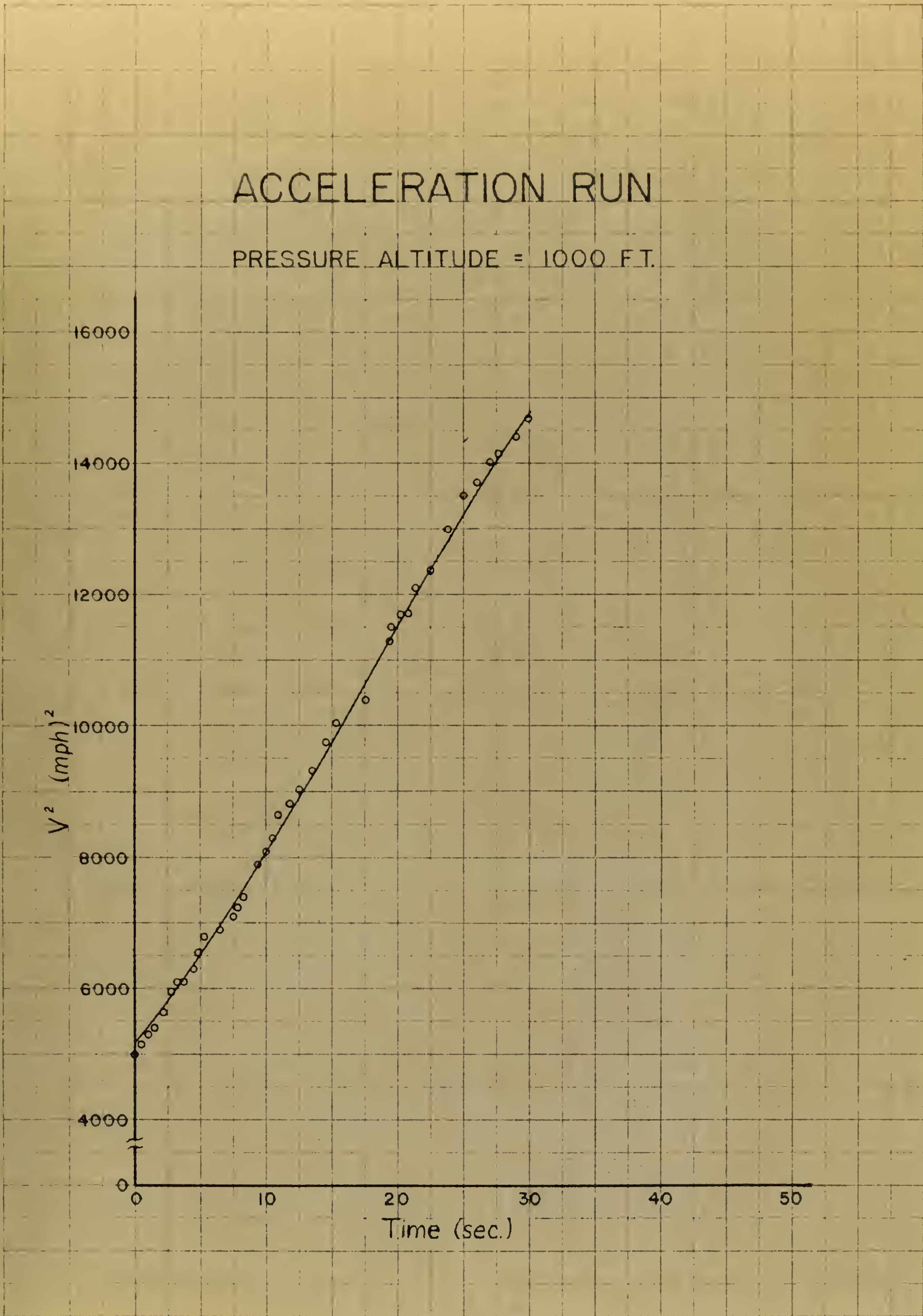
ACCELERATION RUN
PRESSURE ALTITUDE = 1000 FT.
16000
14000
12000
10000
8000
6000
4000
0
V^2 $(mph)^2$
0
10
20
30
40
50
Time (sec.)

Figure 5

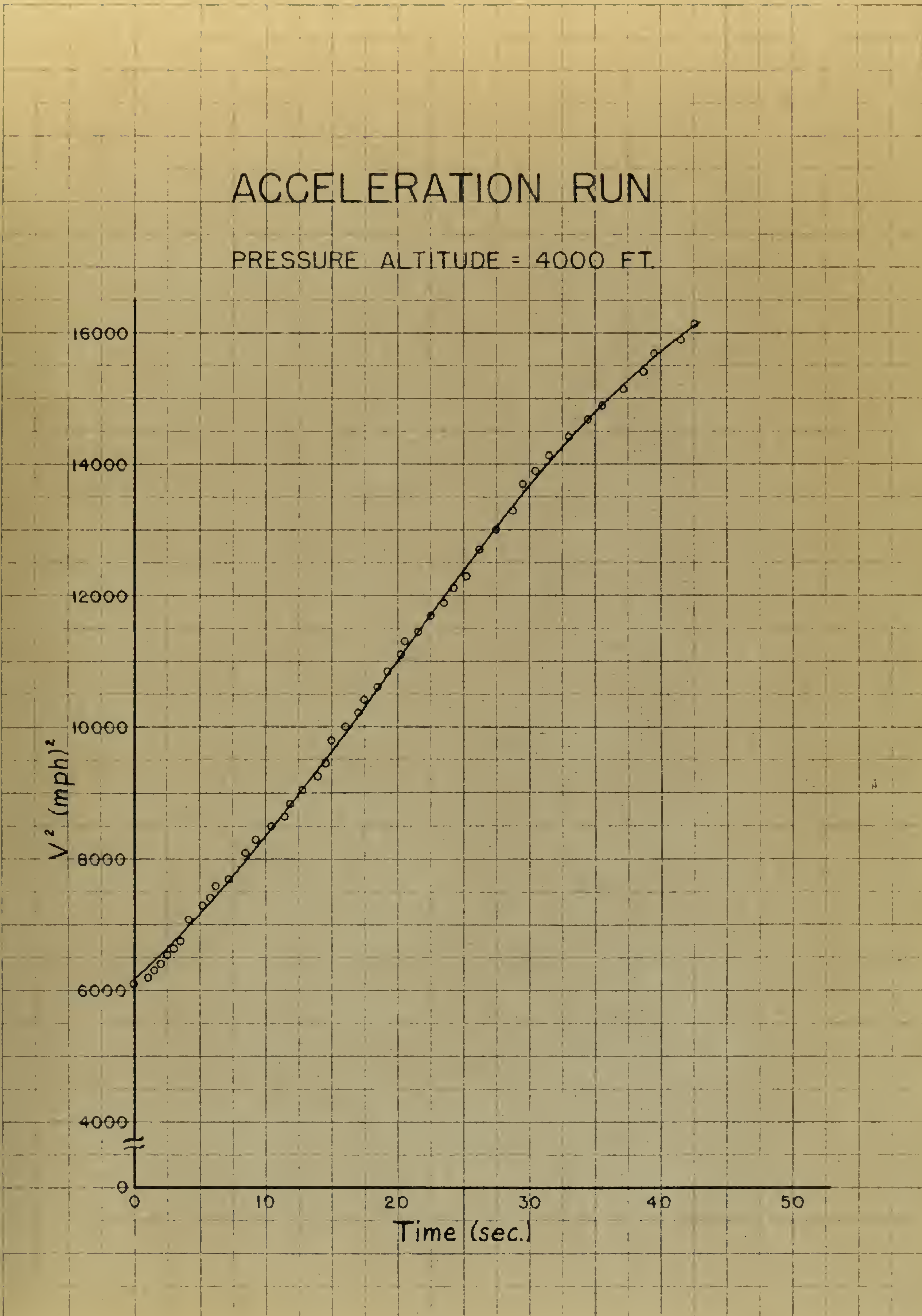
ACCELERATION RUN
PRESSURE ALTITUDE = 4000 FT.
V^2 (mph)^2
16000
14000
12000
10000
8000
6000
4000
0
0
10
20
30
40
50
Time (sec.)

Figure 6

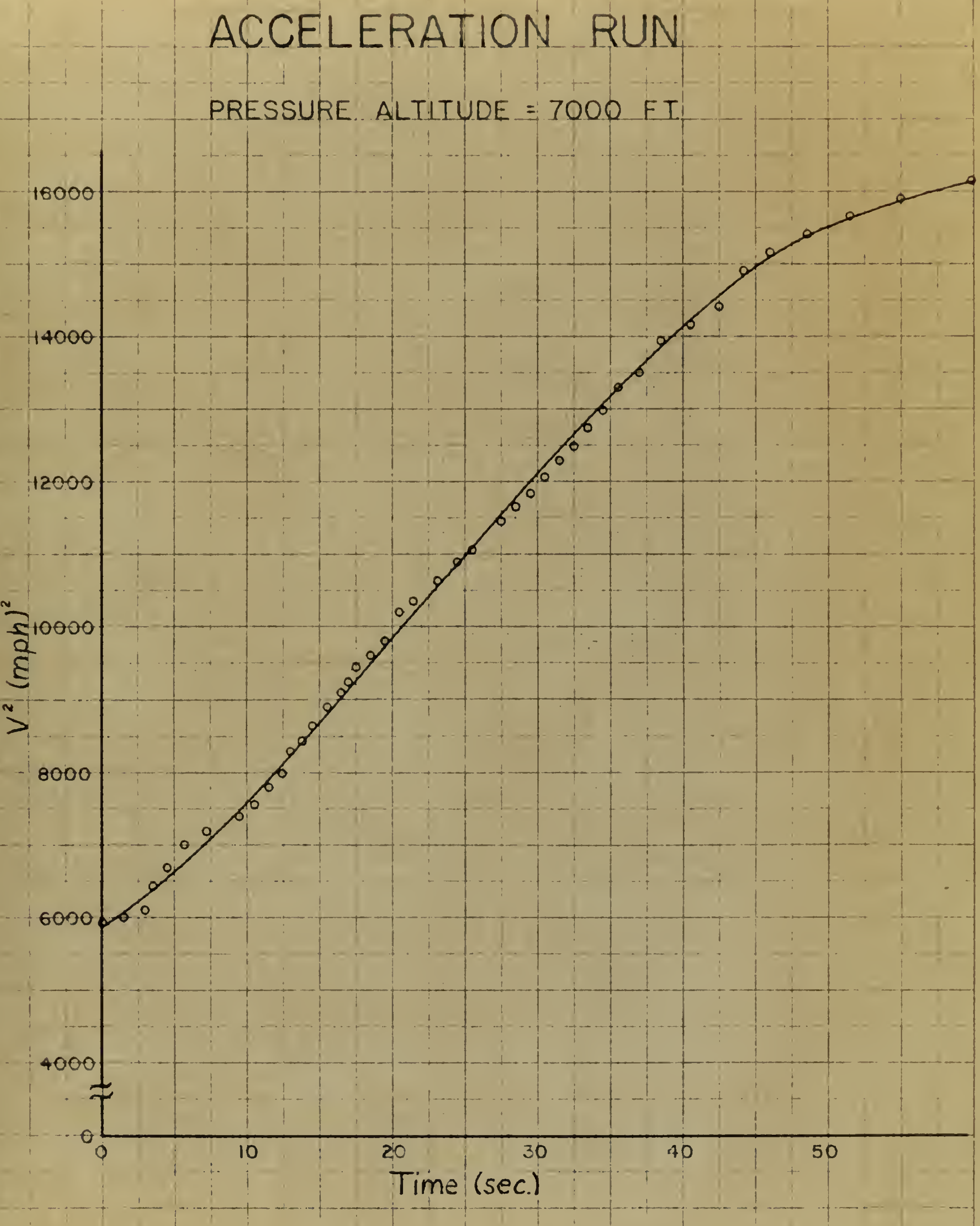
ACCELERATION RUN
PRESSURE ALTITUDE = 7000 FT.
16000
14000
12000
10000
8000
6000
4000
0
V^2 (mph)^2
0
10
20
30
40
50
Time (sec.)

Figure 7

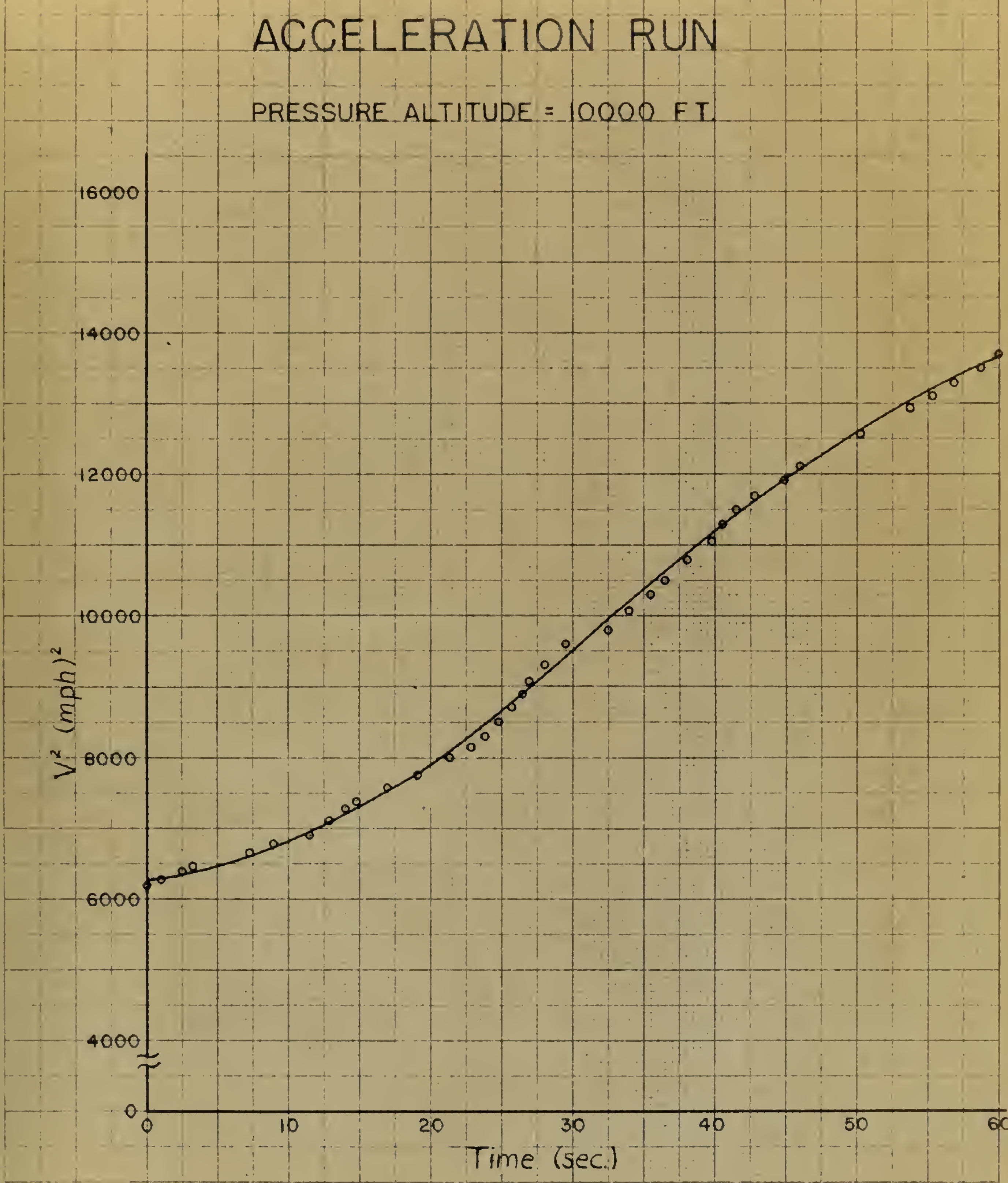
ACCELERATION RUN
PRESSURE ALTITUDE = 10000 FT.
16000
14000
12000
10000
8000
6000
4000
0
V^2 (mph)^2
0
10
20
30
40
50
60
Time (sec.)

Figure 8

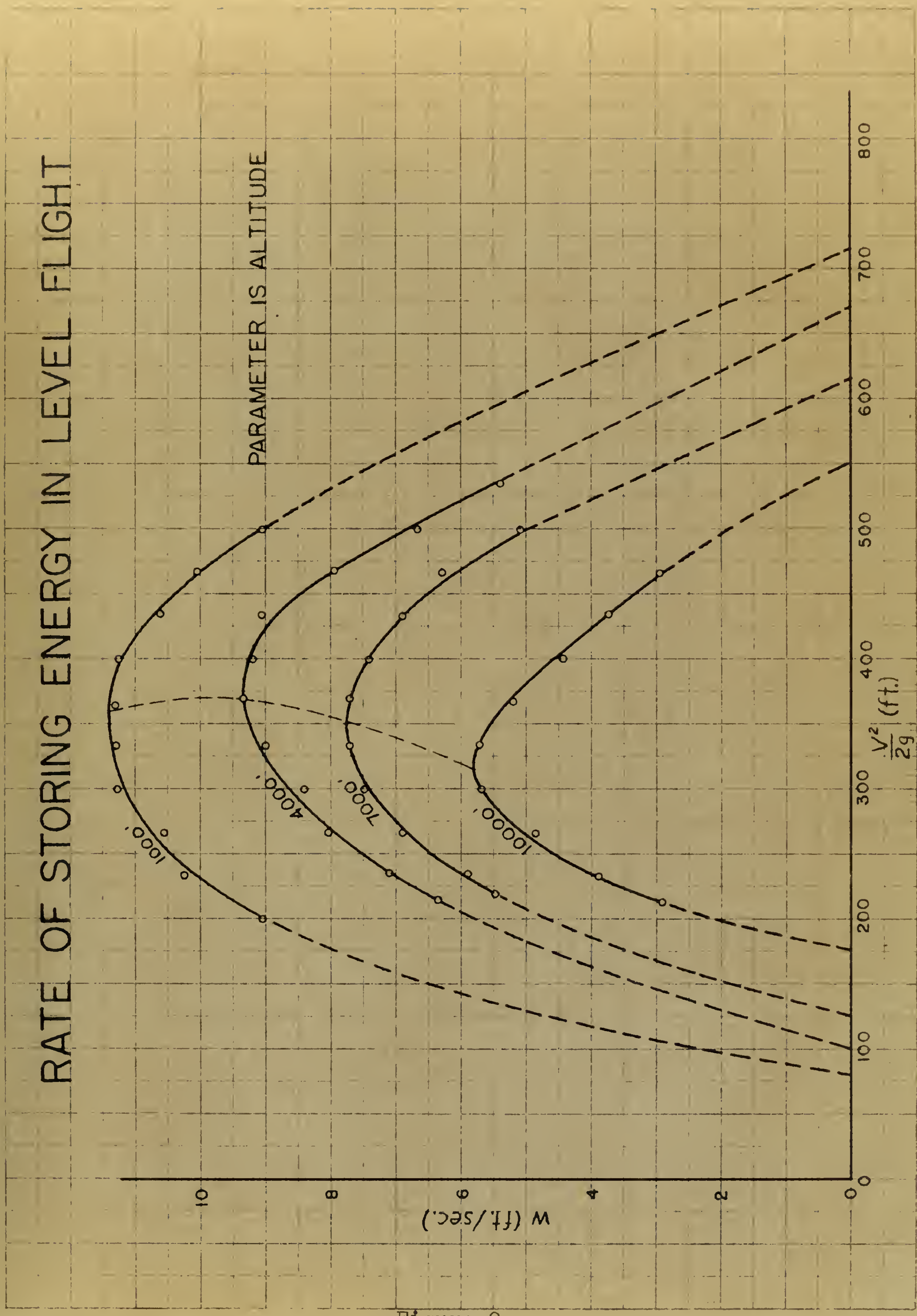
RATE OF STORING ENERGY IN LEVEL FLIGHT
PARAMETER IS ALTITUDE
1000'
4000'
7000'
10000'
w (ft./sec.)
0
2
4
6
8
10
0
100
200
300
400
500
600
700
800
$\frac{V^2}{2g}$ (ft.)

Figure 9

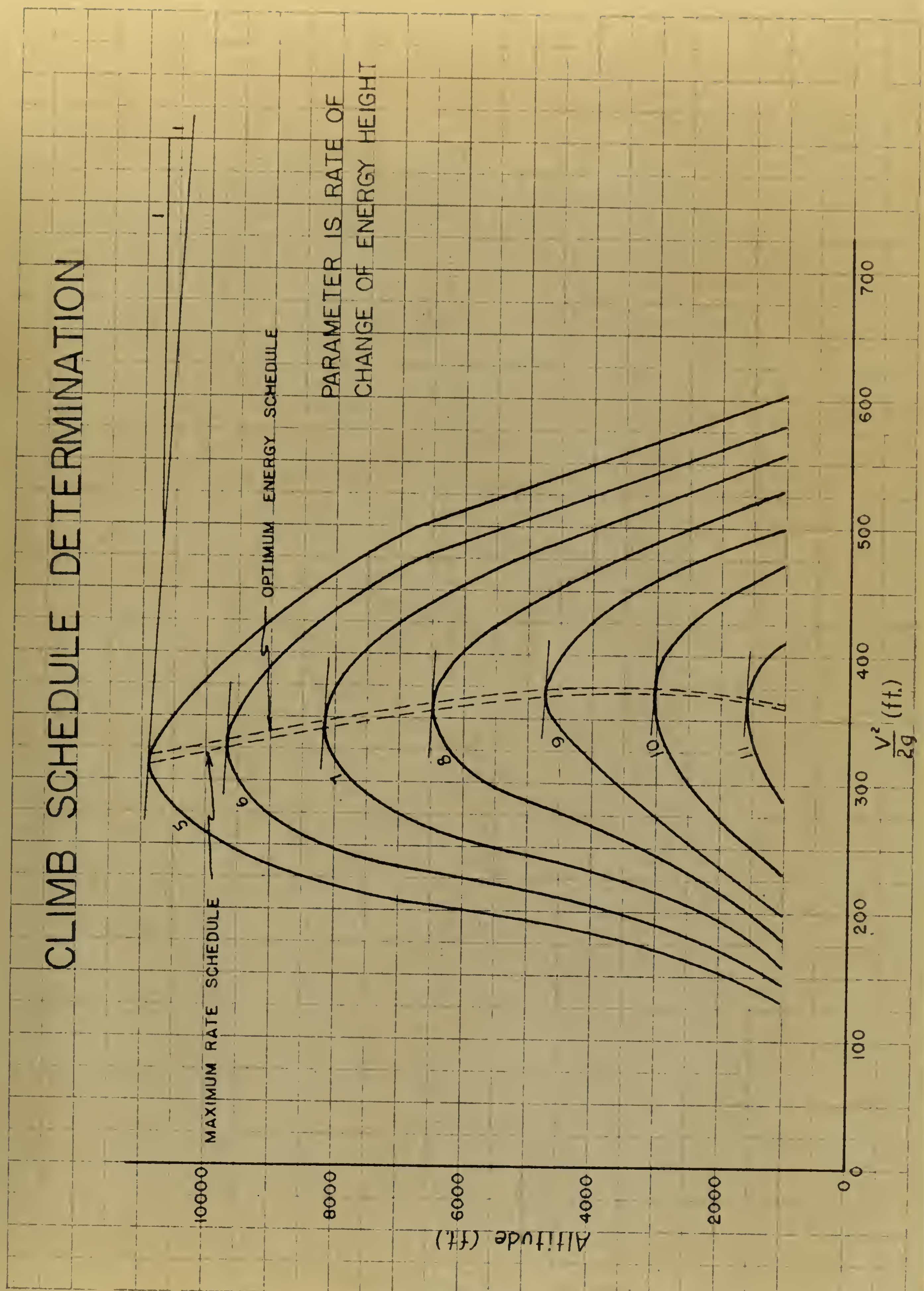
CLIMB SCHEDULE DETERMINATION
MAXIMUM RATE SCHEDULE
OPTIMUM ENERGY SCHEDULE
PARAMETER IS RATE OF
CHANGE OF ENERGY HEIGHT
Altitude (ft)
10000
8000
6000
4000
2000
0
0
100
200
300
400
500
600
700
$\frac{V^2}{2g}$ (ft)
5
6
7
8
9
10
11

Figure 10

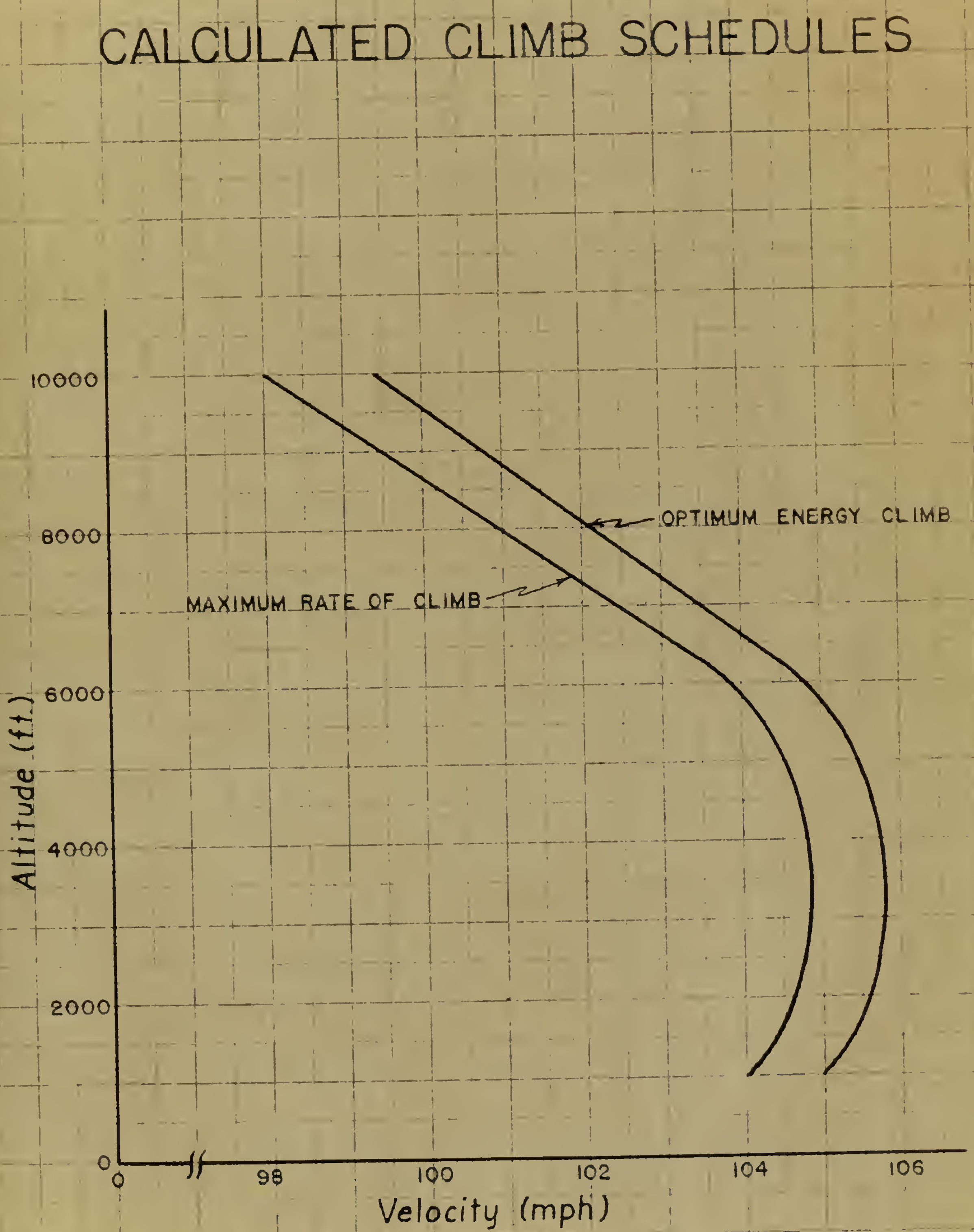
CALCULATED CLIMB SCHEDULES
OPTIMUM ENERGY CLIMB
MAXIMUM RATE OF CLIMB
10000
8000
6000
4000
2000
0
Altitude (ft.)
0
98
100
102
104
106
Velocity (mph)

Figure 11

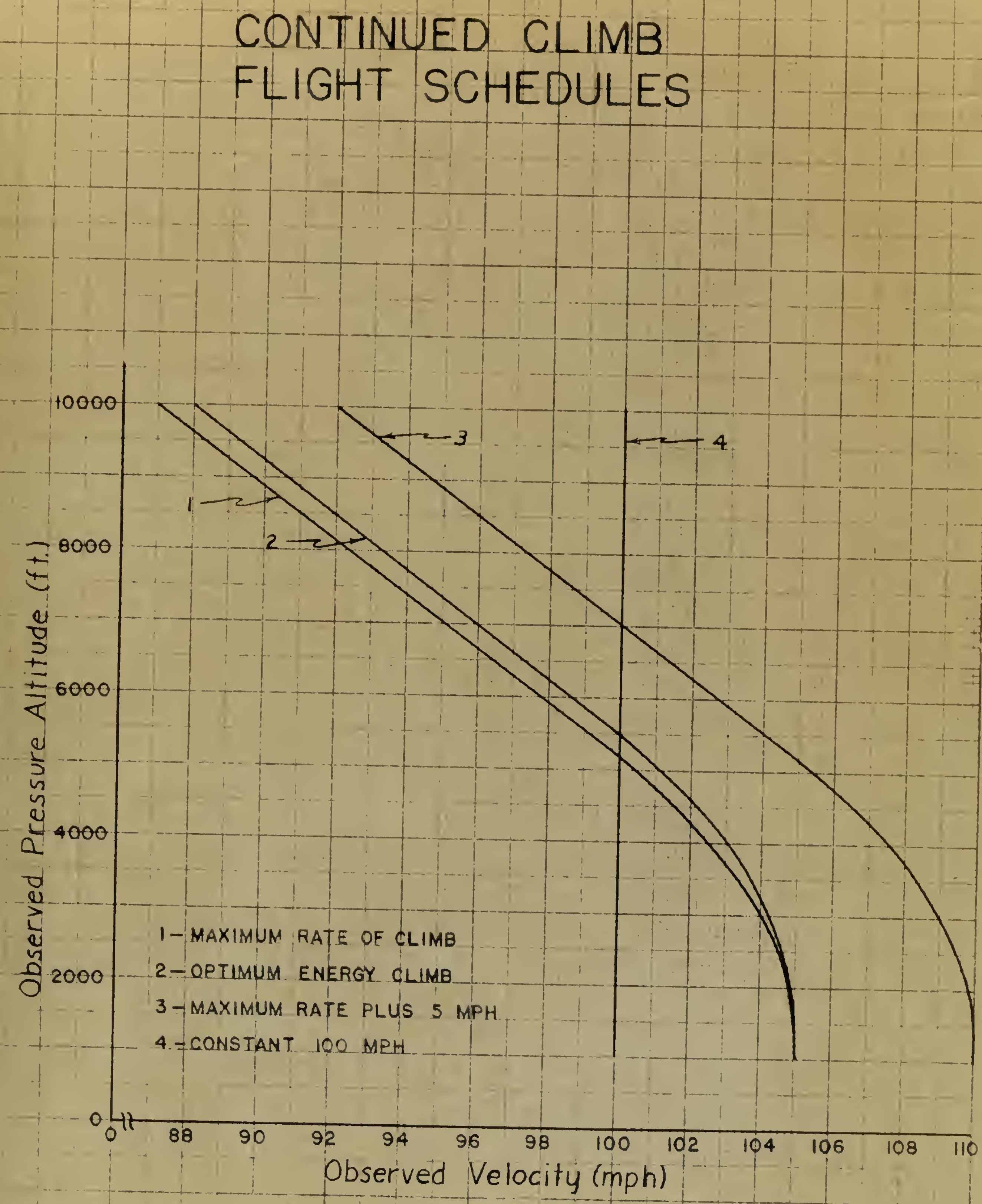
CONTINUED CLIMB
FLIGHT SCHEDULES
Observed Pressure Altitude (ft.)
10000
8000
6000
4000
2000
0
1
2
3
4
1 — MAXIMUM RATE OF CLIMB
2 — OPTIMUM ENERGY CLIMB
3 — MAXIMUM RATE PLUS 5 MPH
4 — CONSTANT 100 MPH
0
88
90
92
94
96
98
100
102
104
106
108
110
Observed Velocity (mph)

Figure 12

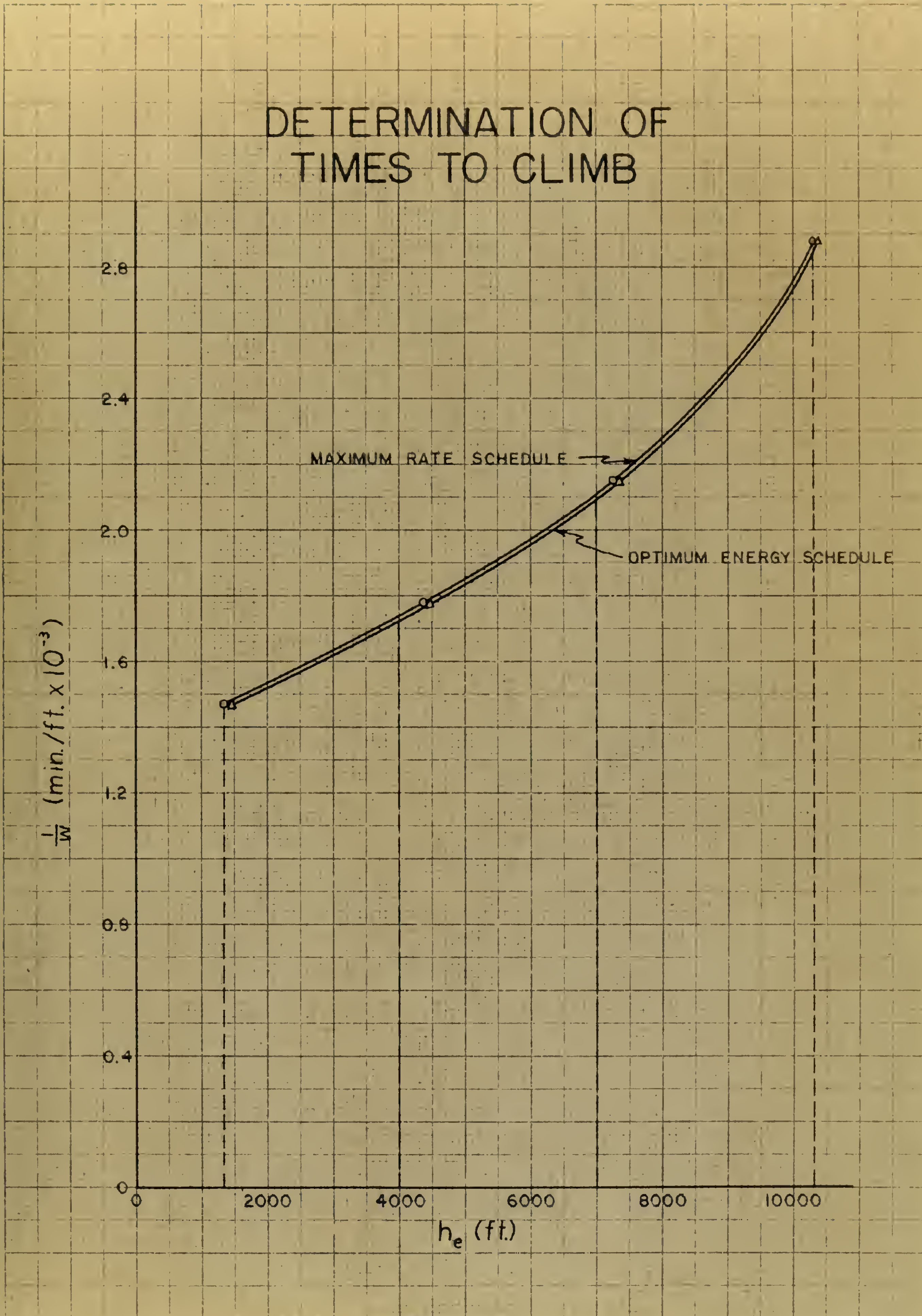
DETERMINATION OF
TIMES TO CLIMB
MAXIMUM RATE SCHEDULE
OPTIMUM ENERGY SCHEDULE
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
$\frac{-1}{\dot{W}}$ (min./ft. x 10^{-3})
0
2000
4000
6000
8000
10000
h_e (ft.)

Figure 13

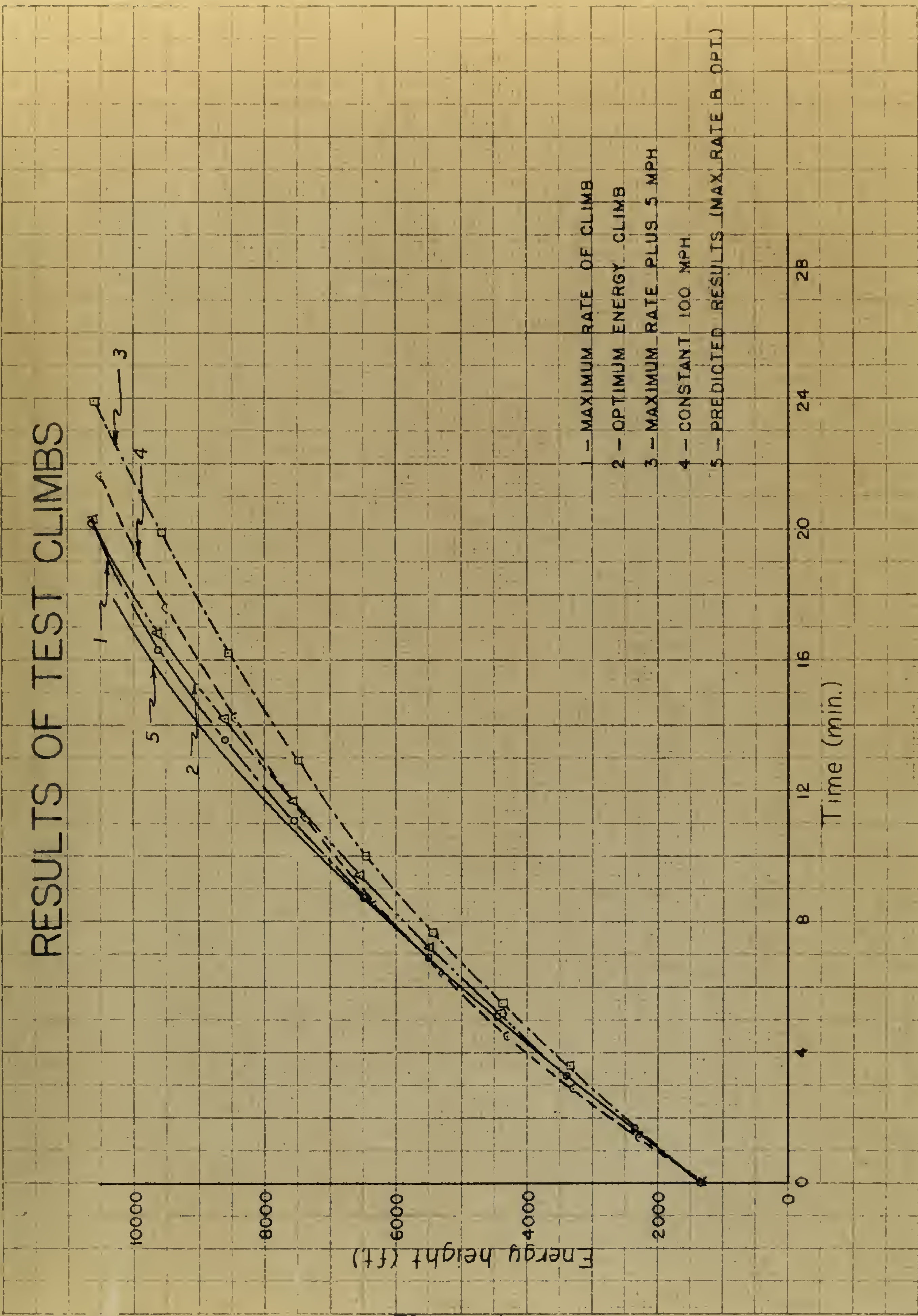
RESULTS OF TEST CLIMBS
1 – MAXIMUM RATE OF CLIMB
2 – OPTIMUM ENERGY CLIMB
3 – MAXIMUM RATE PLUS 5 MPH
4 – CONSTANT 100 MPH
5 – PREDICTED RESULTS (MAX. RATE & OPT.)
Energy height (ft)
0
2000
4000
6000
8000
10000
Time (min.)
0
4
8
12
16
20
24
28
1
2
3
4
5

Figure 14

AIRCRAFT SPECIFICATIONS

Model	North American "Navion"
Length	27.25 ft.
Wing span	33.36 ft.
Gross weight	2750 lb.
Power plant	Continental, six cylinder, horizontal-opposed, 205 hp.
Propeller	Hartzell, selective pitch
Landing gear	Tricycle, retractable
Wing area	184.2 ft^2.
C.G. position	27% MaC

SYMBOLS AND DEFINITIONS

Symbol	Definition	Standard Units
E	Total energy	ft.-lbs
g	Acceleration of gravity	32.2 ft./sec.^{2}
h	Tapeline altitude	ft.
h_e	Energy height	ft.
h_p	True pressure altitude	ft.
h_{p_o}	Observed pressure altitude	ft.
h_{p_i}	Indicated pressure altitude	ft.
KE	Kinetic energy	ft.-lbs.
PE	Potential energy	ft.-lbs.
t	Time	sec., min.
T	Ambient air Temperature	deg. K
V	True airspeed	ft./sec.,mph
V_i	Indicated airspeed	mph
V_{cal}	Calibrated airspeed	mph
V_{max}	Maximum level flight airspeed	mph
V_o	Observed airspeed	mph
V_{o_s}	Standard observed airspeed	mph
W	Airplane gross weight	lbs.
w	Rate of change of energy height (dh_e/dt)	ft./sec.
ρ	Air density	slugs/ft.^{3}
ρ_o	Standard sea level density	.002378 slugs/ft.^{3}
ρ_s	Standard density	slugs/ft.^{3}
σ	Density ratio (ρ/ρ_o)	---
σ_s	Standard density ratio (ρ_s/ρ_o)	--

p	Air pressure	lbs./ft.^{2}
P	Power	ft.-lbs./sec.
S	Wing area	ft.^{2}

Thesis
S715
Stewart
28852
Application of the energy concept to the climb performance of a light propeller-driven airplane.